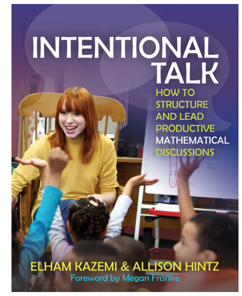
From Intentional Talk (2014) (Stenhouse Publishers)
Kazemi & Hintz.



Classroom discussions are guided by four principles:

1. Discussions should achieve a mathematical goal, and different types of goals require planning and leading discussions differently.
Mathematical Discussions focus on *Concepts, Procedures, Representations*, and *Explanations* and the interrelationships of these foci.

	Goal
Open Strategy Sharing (CH 2)	To have students share as many different ideas as possible in the discussion so they see a range of possibilities, that often leads to Targeted Discussion:
Targeted Sharing:	
Compare and Connect (CH 3)	To compare similarities and differences among strategies
Why? Let's Justify (CH 4)	To generate justifications for why a particular mathematical strategy works/makes sense
What's Best and Why? (CH 5)	To determine a best (most efficient) solution strategy in a particular circumstance
Define and Clarify (CH 6)	To define and discuss appropriate ways to use mathematical models, tools, vocabulary, or notation
Troubleshoot and Revise (CH 7)	To reason through which strategy produces a correct solution or figure out where a strategy went awry

2. Students need to know what and how to share so their ideas are heard and are useful to others.
a. We can use and offer **sentence starters** that *cue students to know what to say*:
"Explain to me what you meant by ______,"
"What would you do if the number was ______?" and
"How is your way different from ______?"
b. We also help students **learn *what to listen for*** so they can contribute to the conversation:
"Listen for how she broke apart the numbers," "Think about whether you are understanding how she used the number line to show her thinking."
c. Similarly, students ***learn how to share*** through our explicit support. *Reinforcing norms* supports students in knowing how to share. For example, you might need to reinforce
where to place oneself : "Stand here so we can see your work"; "Sit knees to knees with your partner so you can listen to each other.";
how loudly to speak: "Speak loudly so everyone can hear your idea"; and
what tools to use: "Use the drawing in your journal to help."

3. Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal.
Teachers can draw attention to the meaningful contributions that all students make and can encourage students to take risks by "assigning competence," or identifying and naming students' specific contributions

4. Teachers must communicate that all children are sense makers and that their ideas are valued.
How we respond to errors and partially developed ideas sends important messages about taking risks. It is not easy for students to express their ideas if there is a high burden to be correct and understand everything the first time around. Being smart in mathematics is not just about speed and accuracy. We want all students in the class to regard themselves as mathematical thinkers and to see themselves as people who can grow and be successful.

Open Strategy Sharing:
In open strategy sharing, students listen for and contribute different ways to solve the same problem. The teacher asks how questions, such as "How did you think about the problem?" and sometimes why questions, like: "Why did you start with the seven?"
Most important, the teacher invites children to share by asking: "Who did it a different way?" Students are oriented to tracking and repeating their classmates' strategies to show they understand what their peers did. The goal of open strategy sharing is to bring out a range of possible ways to solve the same problem and build students' repertoire of strategies.

Class Norms for Open Sharing.
In class we will:

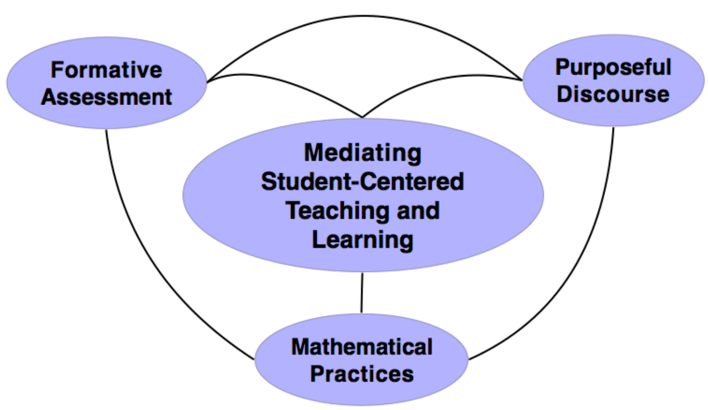
- Make sense of mathematics
- Keep trying even when problems are challenging
- Remember that it's okay to make mistakes and revise our thinking
- Share our mathematical ideas with our classmates (whether we are using words, numbers, pictures, gestures, or tools)
- Listen to understand someone else's idea; give each other time to think
- Ask questions that help us better understand the mathematics
- Agree and disagree with mathematical ideas, not with each other
- Remember that everyone has good mathematical ideas

Talk Moves to Support Classroom Discussions	
Revoicing *"So you're saying . . ."*	• Repeat some or all of what the student has said, then ask the student to respond and verify whether or not the revoicing is correct. Revoicing can be used to clarify, amplify, or highlight an idea.
Repeating *"Can you repeat what she said in your own words?"*	• Ask a student to repeat or rephrase what another student said. • Restate important parts of complex idea in order to slow the conversation down and dwell on important ideas.
Reasoning *"Do you agree or disagree, and why?"* *"Why does that make sense?"*	• After students have had time to process a classmate's claim, ask students to compare their own reasoning to someone else's reasoning. • Allow students to engage with each other's ideas. • Student: "I respectfully disagree with that idea because . . ."; "This idea makes sense to me because . . ."
Adding On *"Would someone like to add on to this?"*	• Prompt students, inviting them to participate in the conversation or to clarify their own thinking. • Student: "I'd like to add on . . ."

Wait Time *"Take your time . . ."*	• Wait after asking a question before calling on a student. • Wait after a student has been called on to give the student time to organize his or her thoughts. • Student: "I'd like more time . . ."
Turn-and-Talk *"Turn and talk to your neighbor . . ."*	• Circulate and listen to partner talk. Use this information to choose whom to call on. • Allow students to clarify and share ideas. • Allow students to orient themselves to each other's thinking.
Revise *"Has anyone's thinking changed?"* *"Would you like to revise your thinking?"*	• Allow students to revise their thinking as they have new insights. • Student: "I thought . . . But now I think . . . because . . ." "I'd like to revise my thinking."

Planning Protocol:

1. (TASK) Pick a problem that can be solved in more than one way. Anticipate what children might do to solve the problem.
2. (WORK FORMS) Decide whether you want children to work on their own, in partners, or in groups to solve the problem.
3. (LAUNCH) Pose the problem, making sure students understand the problem and have a way to get started. (EXPLORE) Take anecdotal records while they work.
4. (SHARE) Have students share out two to four different ways to solve the problem. Use talk moves (see table above) and clear representation to help students understand what they hear.
5. (SUMMARIZE) Close by highlighting the different ways students thought about the problem.

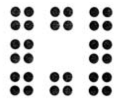


Appendix A: Planning Template for Open Strategy Sharing Discussion

Open Strategy Sharing		
Problem to pose		
Why I chose this problem		
Opening the lesson		
How might my students solve this problem?	**Who solved it this way?**	**Who should share today?**
Notes to myself about what I'm looking for		
Other strategies that emerged during the lesson		
Closing the lesson		

Example Open Strategy Plan (grade 4 multiplication)

Open Strategy Sharing

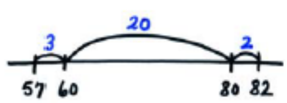
Problem to pose Pose quick image 		
Why I chose this problem	Applying multiplication strategies to count all; seeing and using groups of dots to count how many	
Opening the lesson	Brief intro to keep eyes on document camera. "Your job is to figure out how many dots. I want to hear how you know."	
How might my students solve this problem?	**Who solved it this way?**	**Who should share today?**
9 fours if all were filled out—imagine the group of 4 missing in the middle. 9 x 4 – 4	Ayoub	Share second
Fours in columns: 3 fours and then 2 and then 3 more 12 + 8 + 12	Divina	Start with this one
Counting fours around the perimeter	Olivia	Share third
Notes to myself about what I'm looking for How are students using groups to figure out how many? I really want students to make sense of each other's ideas.		
Other strategies that emerged during the lesson		
Closing the lesson	Reinforce that there are different ways to decompose and see the image.	

Targeted Discussion
COMPARE AND CONNECT

- **Decide which strategies** you want your students to compare and connect.
- **Identify connections** that you believe are important for students to notice between the two or more strategies.
- On your planning sheet, **write out the strategies** like you imagine they will be recorded on the board. (Add more columns to the planning sheet if you are comparing more than two strategies.)
- **Anticipate** what students may notice as they compare and connect the strategies and how you might respond to support their ideas.
- Jot a note to yourself about **the mathematical idea** you want to target during the discussion and highlight at the end of the discussion. Put the note in your pocket so you can quickly remind yourself during the discussion.

As you facilitate the discussion, **stay focused on the targeted strategies and the key mathematical idea**. It can be tempting to pursue other interesting ideas that may emerge (as we do in an open strategy share); however, *a Compare and Connect discussion is all about delving into the connections between the strategies of focus*.

Grade 3 Example:

Compare and Connect	
Strategy 1	**Strategy 2**
Using a number line to compute 82 – 57 	Using a hundreds chart to compute 82 – 57 
What connections are important for students to notice? Because the hundreds chart wraps the numbers, keeping track of jumps means moving from right to left when going backward. On the number line, you can keep track of jumps on the arcs above the numbers.	
Supporting students' thinking	
What students might notice	**How I might respond to support their thinking**
Seeing the jumps is easier on the number line.	How can we mark up the hundreds chart to show the jumps?
You can use the same strategy on both the number line and the hundreds chart.	What's the same? What's different? Which direction do you go on the hundreds chart if you go backward?
The numbers are all marked on the hundreds chart. You use only some numbers on the number line.	How do those differences help you keep track?
What is the key mathematical idea I want to highlight? The jumps on the number line can be mapped onto to the jumps on the hundreds chart.	

You may want to have a Compare and Connect discussion in these situations:

- The problem can be solved in more than one way, and you know, based on your students, that they will have a variety of ways to approach it.
- You want to support your students in making sense of the different strategies that they have generated in order to make sure students don't see the mathematics in the solutions as disconnected.
- You're prompting students along to a slightly more sophisticated strategy.
- You want to compare the use of two different mathematical tools or representations to solve the problem.

Reflect on these questions:
1. When do you think a Compare and Connect discussion would be most useful in a unit you are about to teach?
2. This discussion structure could be useful in helping students see connections between their invented strategies and standard algorithms. How might this discussion structure help students really make sense of the notation in standard algorithms?
3. What kinds of anchor charts or displays could you keep after a Compare and Connect discussion to support students' work as you move through a unit?

Appendix B: Planning Template for Compare and Connect Discussion

Compare and Connect	
Strategy 1	**Strategy 2**
What connections are important for students to notice?	
Supporting Students' Thinking	
What students might notice	**How I might respond to support their thinking**
What is the key mathematical idea I want to highlight?	

Targeted Discussion
WHY? Let's JUSTIFY

Planning for a *Why? Let's Justify* discussion requires us

- To identify the main idea or generalization we want students to examine.
- To anticipate and think through how the justification will sound as students share ideas over the course of the discussion.
- To listen for procedural descriptions that students might give and be sure not to have the conversation stop there.
- To support students in justifying their ideas, when we press beyond procedural explanations into explanations that include reasoning.

Forms of Justification:

1. Justification through appeal to authority (teacher, parent, book, internet)
2. Justification through examples (counter examples, and non-examples; Concept Development Model; inductive reasoning)
3. Justification through a generic example (from specific examples to a generic one that captures all prior specific examples; inductive reasoning; abstracting)
4. Justification through deductive argument (e.g showing that when the commutative property of addition works for 2 numbers, it works for the addition of any amount of numbers).

During a Why? Let's Justify discussion, the talk narrows upon a general claim in order to closely examine the mathematics and generate a justification for it. Certain types of mathematical ideas lend themselves to a discussion that generates a justification for that idea. You may want to have a *Why? Let's Justify* discussion in these situations:

- A rule or "trick" is commonly used, but students may not have a conceptual understanding of why that rule works and therefore may struggle to generalize the rule with accuracy when solving new problems.
- You can connect a strategy students are beginning to use to a visual model or a problem context in order to make sense of how a strategy works regardless of the numbers. For example, when you want to add two-digit numbers, you can always combine the tens and combine the ones separately and then add them together. The model or context serves as a resource for children to verify and test their justification for why the strategy works.

We want to share with you the guidelines that Susan Jo Russell, Deborah Schifter, and Virginia Bastable (2011) provide about how to focus students' attention on justifying a general claim:

1. Choose accessible numbers when first trying to make sense of a general idea.
2. Use a set of expressions or a true/false equation (like you read about in this chapter's vignettes) and focus on the meaning of the expressions instead of just carrying out the computation.
3. Ask students to show their ideas using cubes, number lines, arrays, story contexts, or other representations they have been working with.
4. Identify general claims worth justifying by listening for the patterns, mathematical relationships, or underlying structure of numbers your students notice as they do mathematics.

Grade 4 Example:

Why? Let's Justify

What mathematical strategy or idea are we targeting in our discussion?

$6 \times 19 = 6 \times 10 + 6 \times 9$
True or false?

Remember to ask "Why?"

What is the explanation I want students to come up with?
(Include sketch of any representations that might be helpful for the explanation.)

Choose either 6 groups of 19 or 19 groups of 6. Stick with one interpretation for today's lesson.

Six groups of 19 is the same as 6 groups of 10 and 6 groups of 9—and be able to use the picture to see, understand, and prove why.

Draw picture on the board so that groups of 10 and groups of 9 are in line. This will help students see groups of 19.

Could also draw a picture of 19 groups of 6. So the picture should be 10 groups of 6 and then 9 groups of 6, which makes 19 groups of 6.

Supporting students' thinking
(If students say this . . . then I may ask them this to work toward stronger justification.)

What students might say	How I might respond
"The numbers are broken up."	"How are the numbers broken?" Establish that it's by place value components.
Read the number sentence using only language of "times."	Ask them to use the language of "groups of." (E.g., 6 groups of 19 is the same as 6 groups of 10 and 6 groups of 9.)

Appendix C: Planning Template for Why? Let's Justify Discussion

Why? Let's Justify	
What mathematical strategy or idea are we targeting in our discussion?	
What is the explanation I want students to come up with? (Include sketch of any representations that might be helpful for the explanation.)	
Supporting students' thinking (If students say this . . . then I may ask them this to work toward stronger justification.)	
What students might say	**How I might respond**

Targeted Discussion
What's Best and Why?

Instead of eliciting many different ways to solve a particular problem, the teacher structures the discussion in one of two ways:
1. Shows a particular strategy and then asks students to generate an effective use of that strategy
2. Shows a few different ways to solve a problem and asks students to figure out which is the most efficient strategy for that problem

With *What's Best and Why?* discussions, we're asking students to analyze situations and decide on the effectiveness of particular strategies. The emphasis is not on generating the strategies themselves but on judging when to use particular approaches.

Grade 3 Example:

What's Best and Why?
What is my goal? What strategy(ies) am I highlighting? To think together about which strategy to choose when subtracting numbers. Specifically, counting back when numbers are far apart. Counting on when numbers are close together.
What tasks/problems help us discuss what is best and why? Number string 33 – 4 33 – 7 42 – 37 33 – 28 • Return to strategies from yesterday. • Ask students to think about why they chose going backward or taking away for first two and adding up for the second two. Perhaps focus just on what they did for 33 – 4 and 33 – 28. • Ask students to reflect on why those strategies work in relation to how the operation of subtraction behaves.
What would I like to hear from my students? When numbers are far apart, it can be easier to go backward from the larger number, to take away. When numbers are close together, it can be easier to go forward from the smaller number, to count on or add up. Either way works because we can think about subtraction as comparing two numbers and seeing how far apart they are from each other or as taking away one amount from another.

Appendix D: Planning Template for What's Best and Why? Discussion

What's Best and Why?
What is my goal? What strategy(ies) am I highlighting?
What tasks/problems help us discuss what is best and why?
What would I like to hear from my students?

Targeted Discussion
Define and Clarify

How do students develop meaning for mathematical tools?

- We take this question to heart as we think about discussions that serve to define and clarify new objects and support students in using them meaningfully.
- Discussing how and why to use mathematical objects is an important part of developing problem-solving skills and number-and-operation sense.
- It is important to consider when Define and Clarify discussions can and should occur (e.g., when objects are first being introduced or when teachers want to help students refine their use).

Explicit conversations about how and when to use these tools and concepts accurately are important components of making mathematical work accessible and meaningful to students

Grade 4 Example:

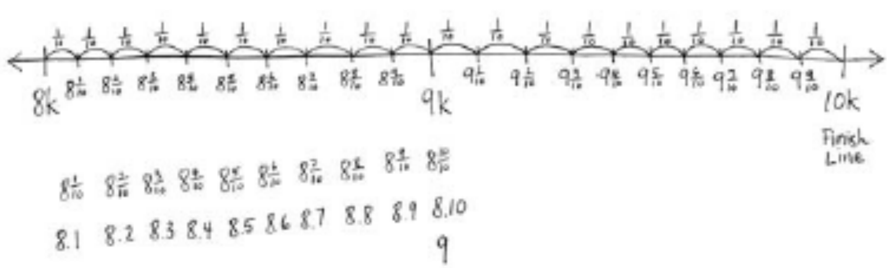
Define and Clarify

What new tool, representation, symbol, or vocabulary are we targeting in our discussion? Is this new to the students or are they using it in a new way?

How to write 8 10/10 as a decimal. Use the idea of 10/10 = 1 to make sense of why the number is not written 8.10 but instead is written 9.0.

What problem or task are we working on? How will I support meaning making? What misconceptions might arise?

Is 8 10/10 written like 8.10 or 9? (Question raised by students.) Return to 10K representation and story context to support making meaning of how to write 10/10.



Grade 5 Example:

Define and Clarify

What new tool, representation, symbol, or vocabulary are we targeting in our discussion? Is this new to the students or are they using it in a new way?

Open array. Representation used for making meaning of multiplication—introducing new usage—to help make sense of division.

What problem or task are we working on? How will I support meaning making? What partial understandings might arise?

How to solve a division problem using what you know about multiplication, or groups. Support making meaning of groups of 10 strategy.

$240 \div 12$

$12 \times 10 = 120$

$12 \times 10 = 120$

$120 + 120 = 240$

How are we building up to the total of 240 with 10 groups of 12 and 10 more groups of 12?

Where is the 240? Where is the answer?

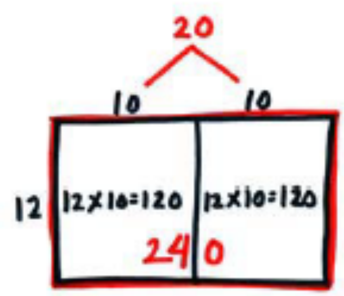


Figure 6.8 This portion of the poster shows a representation of multiplying up to solve 240 ÷ 12.

$240 \div 12$

$12 \times 10 = 120$

$12 \times 10 = 120$

$120 + 120 = 240$

$240 \div 12 = 20$

Appendix E: Planning Template for Define and Clarify Discussion

Define and Clarify
What new tool, representation, symbol, or vocabulary are we targeting in our discussion? Is this new to the students or are they using it in a new way?
What problem or task are we working on? How will I support meaning making? What partial understandings might arise?

Targeted Discussion:
Troubleshoot and Revise.

A *Troubleshoot and Revise* discussion can be initiated by the teacher or by one or more students. In either case, the teacher or student has noticed something is awry and seeks the collective engagement of the class to figure out what needs to be revised.

A *Troubleshoot and Revise* discussion can be initiated for a variety of reasons.

- Students might recognize that they have partial understandings and are stuck.
- They might notice that a strategy they used, which seemed to make sense, resulted in an answer different from that of their classmates.
- The teacher might also notice a misunderstanding that is bubbling up and worthy of attention.

We want students to know that thoughtful mathematicians voice their confusions; thinking collaboratively through errors can help everyone better understand the mathematics. We want to frame mistakes as "desirable contributions."

A *Troubleshoot and Revise* discussion is an opportunity for a student or group of students to think aloud together about mathematics that may be puzzling and, using the ideas of others, engage in collaborative sense making. Sometimes a confusion is minimal and may not warrant a group discussion. Other times, the confusion is greater, such as when students are mixing up important mathematical understandings or when an error is prevalent among the students in a class. A targeted discussion is useful in slowing down the conversation to uncover the confusion and bring new understanding. You may want to have a *Troubleshoot and Revise* discussion in these situations:

- You observe several students in your class grappling with an idea, and you think other students could help clarify the confusion.
- A student comes to you with an idea he or she is willing to put in front of the class, and you believe the class has the resources to support the student in thinking through the idea.
- You want to support norms for revising one's thinking.

Grade 4 Example:

Troubleshoot and Revise
What is the confusion or misunderstanding we will discuss and revise? Jason wants help explaining why this statement is false: $80 \div 4 = (80 \div 2) + (80 \div 2)$
What is the insight I'd like students to understand? The left side of the equation is about dividing 80 by 4. The right side of the equation is about dividing 160 by 2, because 2 sets of 80 are each being divided by 2.
Problem context, diagrams, or questions that might be useful to use during the discussion • Ask students to consider a problem context as a way of supporting Jason to think through the problem: Is 80 Skittles divided by 4 people the same as 80 Skittles divided by 2 people and another 80 Skittles divided by 2 people? • Use turn-and-talk for students to first generate some ideas. Monitor turn-and-talk and select students who might be able to use the problem situation to determine whether the equation is true or false. • Use a diagram to support visualizing what the equation is saying. • Check in with Jason as explanations are provided to see if he is developing new insights. Ask him to state what his new insights are.
Exit ticket Ask everyone to explain why the statement we focused on is false.

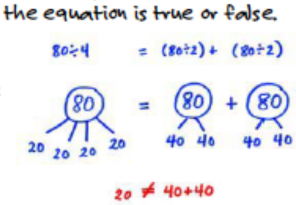


Figure 7.9 This poster provides sentence stems students can use during troubleshooting discussions.

Troubleshooting

<u>For asking my classmates to help me think:</u>
I am not sure about something and I want to ask for ideas.
Can you help me understand why ______?
I want to revise my thinking.

<u>For helping my classmates think:</u>
Can you tell us more about what is confusing you?
What part feels the most confusing to you?
What do you understand about ______?
What do you know that can help you think about this?

Appendix F: Planning Template for Troubleshoot and Revise Discussion

Troubleshoot and Revise
What is the confusion or misunderstanding we will discuss and revise?
What is the insight I'd like students to understand?
Problem context, diagrams, or questions that might be useful to use during the discussion
Exit ticket